

STELLAR EVOLUTION AT HIGH MASS INCLUDING THE EFFECT OF A STELLAR WIND

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ABSTRACT

Evolutionary tracks for stars of 15–120 M_{\odot} undergoing mass loss due to a stellar wind have been computed from the zero-age main sequence to the end of core helium burning. All the models are based on Cox-Stewart opacities, and take fully into account semiconvective and convective modifications of the interior structure. Various cases of mass loss have been considered. If the amount of mass loss on the main sequence is assumed to be small, the post-main-sequence tracks turn out to be very sensitive to whether the Schwarzschild or the Ledoux criterion for convection is adopted, as well as to what values are adopted for the initial metals abundance and convective mixing length. A somewhat larger assumed amount of main-sequence mass loss invariably will produce a red supergiant (due to the suppression of hydrogen-shell convection), while a very much larger assumed amount of mass loss during either the main-sequence or post-main-sequence stages will always produce a blue helium star (due to removal of most of the hydrogen envelope). Comparison of the theoretically derived results with observations of OBN stars, WN stars, and bright supergiants suggests that the most massive stars could, in the course of their lifetimes, lose a substantial amount of mass. However, the interpretation for stars of slightly lower initial mass is rather more ambiguous, except that mass loss is probably not very important for initial masses below $\sim 30 M_{\odot}$.

Subject headings: stars: early-type — stars: evolution — stars: interiors — stars: winds

1. INTRODUCTION

Stars of high luminosity are observed to be shedding their atmospheres at a rate which may be rapid enough to have important evolutionary consequences. In an early theoretical paper, Tanaka (1966*a*) specified the rate of mass loss and then followed the consequences for evolution in the case of a very massive star on the main sequence. His basic conclusions were that, compared with a star conserving its mass, a mass-losing star at the same stage of central hydrogen depletion possesses (1) a lower luminosity; (2) a lower effective temperature; (3) a reduced convective instability in the (mostly radiative) envelope; (4) a larger fraction of its mass occupied by the convective core, despite the fact that the mass of the convective core itself is smaller; and (5) an increased hydrogen-burning lifetime. In an important sequel to his first paper, Tanaka (1966*b*) demonstrated that once the hydrogen-processed layers are exposed at the surface, the effective temperature of the star begins to increase. Without exception, all subsequent work on the subject has essentially verified Tanaka's basic conclusions (Hartwick 1967; Simon and Stothers 1970; Chiosi and Nasi 1974; de Loore, De Grève, and Lamers 1977; Dearborn and Eggleton 1977; Dearborn *et al.* 1978; Sreenivasan and Wilson 1978; Chiosi, Nasi, and Sreenivasan 1978; de Loore, De Grève, and Vanbeveren 1978; Stothers and Chin 1978; Czerny 1979).

Given the observational uncertainty of the rate of mass loss, authors who have calculated stellar models

have simply followed Tanaka's procedure of specifying it, in one way or another. Then, with the rate of mass loss as a free parameter, sets of evolutionary tracks can be plotted on the H-R diagram and compared with the observations. Particularly difficult to explain have been: (1) the remarkably continuous distribution of stars between spectral types O and early A for luminosities in the range $4.5 < \log (L/L_{\odot}) < 5.3$; (2) the strong concentration of stars toward much earlier spectral types at the brighter luminosities; (3) the absence of M supergiants with luminosities greater than $\log (L/L_{\odot}) \approx 5.3$; and (4) the peculiar properties of the single Wolf-Rayet and OBN stars. It has turned out that the assumption of an extremely high rate of mass loss from stars initially more massive than $\sim 30 M_{\odot}$ can alleviate some of these difficulties (see the papers cited above). But it is more than likely that only a moderate rate of mass loss characterizes most main-sequence stars up to $\sim 60 M_{\odot}$. As a consequence, post-main-sequence mass loss may be the determining factor in the evolution of the majority of massive stars.

A few fragmentary studies of post-main-sequence evolution with mass loss have been undertaken for stars of high mass (Hayashi, Hōshi, and Sugimoto 1962; Hartwick 1967; Simon and Stothers 1970; Bisnovatyi-Kogan and Nadezhin 1972; Chiosi and Nasi 1974; Sreenivasan and Wilson 1978; Chiosi, Nasi, and Sreenivasan 1978; Stothers and Chin 1978). From these studies, two important conclusions seem to be well established. First, if previous mass

loss was extraordinarily heavy, the star permanently maintains a high effective temperature. Second, if previous mass loss was only moderately heavy, the star evolves quickly into a cool supergiant, whereupon it either sheds most of its remaining hydrogen envelope and becomes a hot helium star, or, by retaining a sufficiently massive hydrogen envelope, it continues to evolve as a very cool supergiant. Little is known of the case (perhaps the most important one) where previous mass loss is assumed to be small.

Because the number of studies has been so limited, many important questions are still unanswered. Although some of these questions have been addressed in our previous paper (Stothers and Chin 1978) where Carson's opacities were adopted for the stellar models, the situation is still very unclear for the Cox-Stewart opacities. Among these questions are: What, precisely, are the effects on the later evolution that are produced by different assumed amounts of main-sequence mass loss? How does continuing mass loss among the cool supergiants affect their evolution? How important is the choice of the initial chemical composition? What is the evolutionary significance of the adopted criterion for convection (Schwarzschild criterion versus Ledoux criterion)? What role is played by wide-scale mixing in the most massive stars? Finally, are the mass-loss rates needed to achieve agreement with observations realistic?

It is these and other questions which, on the basis of stellar models constructed with Cox-Stewart opacities, we intend to study in the present paper. In § II, our basic assumptions about the rates of mass loss are stated. In §§ III and IV, evolutionary sequences with semiconvective mixing based on the Schwarzschild criterion and on the Ledoux criterion are treated separately, while in § V various hypotheses about the extent of mixing in the most massive stars are examined. Then the collected theoretical results are confronted with observational data in § VI, which is followed by a brief summary of the paper in § VII.

II. ASSUMPTIONS

To illustrate the various consequences of mass loss, we have investigated the following four possibilities, in conformity with the treatment in our earlier paper (Stothers and Chin 1978):

Case A.—No mass loss occurs at all.

Case B.—Mass loss occurs at all stages of evolution in accordance with an assumed rate

$$-dM/dt = kLR/M. \quad (1)$$

With L , R , and M expressed in solar units, k can be specified in units of $M_{\odot} \text{ yr}^{-1}$. For most of our evolutionary sequences we have adopted $k = 1 \times 10^{-11}$.

Case C.—Mass loss occurs only for $\log T_e < 3.85$, i.e., only among late-type supergiants. The rate is given by equation (1) with $k = 1 \times 10^{-11}$.

Case D.—Sudden mass outflow occurs when the star becomes a yellow supergiant. Much uncertainty, however, attends the basic assumptions involved. We

have adopted $\log T_e = 3.7$ as the critical effective temperature at which outflow begins; $20 M_{\odot}$ as the smallest initial stellar mass to have outflow possible; and whatever maximum rate of mass loss our computer program can handle (typically, $\sim 10^{-2} M_{\odot} \text{ yr}^{-1}$).

Mass has been removed from the stellar models in accordance with the prescription given by Kippenhahn and Weigert (1967). Like those authors, we have here neglected entropy changes in the outer layers of the models, since, although they are sometimes large, these changes have been found not to create any significant differences in the characteristics of computed evolutionary sequences (Stothers and Chin 1978), while, on the other hand, they greatly increase the necessary computation time.

The adopted criterion for convective neutrality (i.e., for semiconvection) is assumed here to be the same as the criterion that governs the outbreak of convective instability. This assumption is in accordance with usual practice. In one set of evolutionary sequences, the Schwarzschild (S) criterion for convection has been adopted, and in a second set the Ledoux (L) criterion has been used. Nonadiabatic convection develops in the present models only at low effective temperatures and in the outer layers of the star; it has been treated with the customary mixing-length theory. However, our previously published stellar models based on the Cox-Stewart opacities employed a mixing length proportional to *density* scale height; here we adopt a mixing length proportional to *pressure* scale height. In all but a few test cases, we have set the proportionality factor α_p equal to 1.

Initial chemical composition parameters are taken to be $(X_e, Z_e) = (0.739, 0.021)$. With starting masses of 15, 30, 60, and $120 M_{\odot}$, we have evolved most of the sequences all the way to the end of core helium burning.

Special notation in the present paper follows our earlier papers. We note: $\log T_e$ (tip), logarithm of the hottest effective temperature that is achieved during the *slow* stages of core helium burning; $\log T_e(b/y)$, logarithm of the transitional effective temperature that divides the slow *blue* stages of core helium burning from the fast *yellow* stages; τ_H , lifetime of core hydrogen burning; τ_{He} , lifetime of core helium burning, as measured from the instant of central hydrogen exhaustion; τ_b/τ_{He} and τ_y/τ_{He} , fractions of the helium-burning lifetime that are spent in the blue stages and yellow stages, respectively. These quantities are listed in Table 1.

III. EVOLUTION BASED ON THE SCHWARZSCHILD CRITERION FOR CONVECTION

a) Case A

Since evolution without mass loss has already been treated by us for stars in the mass range $15\text{--}60 M_{\odot}$ (Stothers and Chin 1976), we shall merely recount here the main evolutionary features necessary for understanding the cases that include mass loss.

When a massive star leaves the zero-age main sequence (ZAMS), semiconvection sweeps through

TABLE 1
EVOLUTIONARY SEQUENCES WITH MASS LOSS FOR STARS OF 15, 30, AND 60 M_{\odot} INITIALLY^a

Initial M/M_{\odot}	Criterion for Convection	Case	$\log T_e$ (tip)	$\log T_e$ (b/y)	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_{He}	τ_y/τ_{He}	Final M/M_{\odot}
15.....	S	A	4.20	~ 3.8	11.992	0.115	0.848	0.036	15.0
	S	B	3.98	~ 3.8	12.296
	L	A	...	3.8^b	11.850	0.092	0.040	0.002	15.0
	L	C	4.56	~ 4.0	11.850	0.092	0.760	0.009	4.4
30.....	S	A	4.24	~ 4.1	6.101	0.086	0.968	0.032	30.0
	S	B	4.19	~ 3.8	5.945	0.089	0.975	0.025	17.5
	L	A	...	4.1^b	5.763	0.082	0.034	0.005	30.0
	L	C	4.67	~ 4.3	5.763	0.083	0.831	0.015	11.3
60.....	L	D	4.81	3.7	5.763	0.082	1.000	...	11.6
	S	A	3.94	~ 3.6	3.830	0.086	1.000	...	60.0
	S	B	4.28	~ 3.8	3.726	0.087	1.000	...	37.6
	L	A, C, D	4.22	~ 4.0	3.709	0.086	1.000	...	60.0

^a $k = 1 \times 10^{-11} M_{\odot} \text{ yr}^{-1}$ (cases B and C); $(X_e, Z_e) = (0.739, 0.021)$.

^b Assumed.

the layers that contain a gradient of mean molecular weight, and slightly alters the hydrogen profile of these layers. This process continues until the star has passed the stage of its coolest effective temperature on the main sequence (TAMS stage), whereupon semi-convection usually disappears. When hydrogen is exhausted at the center of the star, the region of greatest convective instability shifts abruptly from the center to the layers immediately above the hydrogen-burning shell, where semiconvection formerly existed. Convection rapidly mixes the chemical composition of these layers, so that the hydrogen profile of the star acquires a local plateau. This large-scale homogenization of the envelope limits the radius expansion of the star, which thus burns core

helium as a blue, rather than a red, supergiant (see Fig. 1).

A fully convective zone (FCZ) of this type appears in stars of as low a mass as $\sim 6 M_{\odot}$, but is considerably larger in stars of higher mass, and tends toward a limiting size in stars more massive than $\sim 45 M_{\odot}$. Above this mass, the very high luminosity of the star is able to expand the envelope against the restoring effect of the FCZ. For this reason, remarkably low effective temperatures are encountered along the helium-burning track for $60 M_{\odot}$.

Before proceeding further, it is desirable to mention an alternative treatment of convective instability in the hydrogen-poor layers. Iben (1966a, b) and Lamb, Iben, and Howard (1976) assumed that *all* the

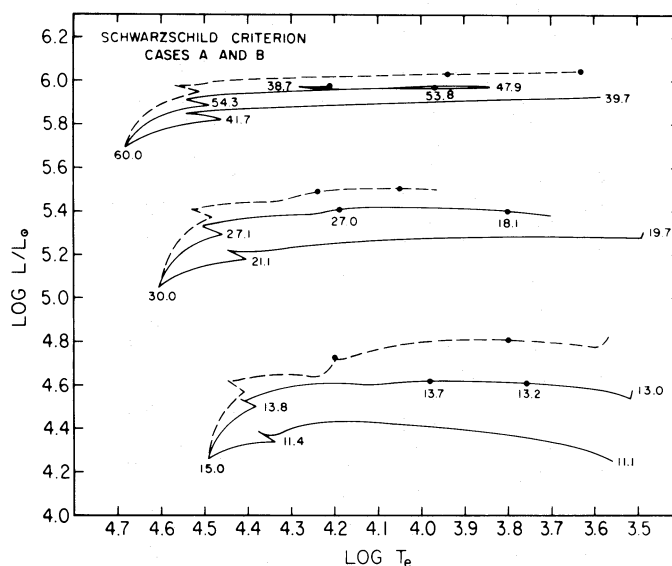


FIG. 1.—H-R diagram showing the evolutionary tracks for stellar models based on the Schwarzschild criterion for convection, according to case A (*dashed lines*) and case B (*solid lines*). In descending order of luminosity, the tracks in each triplet represent $k = 0$, 1×10^{-11} , and 3×10^{-11} . Heavy dots mark the beginning and end of the slow blue stages of core helium burning. Evolution in the red-supergiant region has not been computed. Masses are indicated in solar units.

TABLE 2
LIFETIME OF CORE HYDROGEN BURNING BASED ON THE
SCHWARZSCHILD CRITERION FOR CONVECTION
IN CASES A AND B

INITIAL M/M_{\odot}	$\tau_H(10^6 \text{ yr})$		
	$k = 0$	$k = 1 \times 10^{-11}$	$k = 3 \times 10^{-11}$
15.....	11.99	12.30	13.23
30.....	6.10	5.94	6.26
60.....	3.83	3.73	3.85

convectively unstable layers are mixed homogeneously. This approach, though perhaps questionable from a physical point of view, does lead to a final hydrogen profile at the time of core helium ignition that is not markedly different from the profile derived by the proper inclusion of semiconvection; thus, it is not surprising that the evolutionary tracks obtained in the two cases differ by an amount (Sreenivasan and Ziebarth 1974; Schlesinger 1975; Chiosi and Nasi 1978) that is relatively small in comparison with other uncertainties. However, stellar masses higher than $30 M_{\odot}$ have not yet been studied with this method.

b) Case B

The general effects of mass loss on the main-sequence phase of evolution have already been enumerated in § I. Evolutionary tracks for our new models are shown in Figure 1. They supplement the tracks computed for this phase of evolution by Chiosi and Nasi (1974) with a similar representation of the mass-loss rate.

One new result derived here concerns the effect of mass loss on the hydrogen-burning lifetime. It is already known that mass loss normally lengthens the lifetime, because the reduction of the star's luminosity is larger than the reduction of its core mass. However, the suppression of semiconvection tends to shorten the lifetime, by eliminating the extra hydrogen fed into the convective core. This effect may be seen in Table 2 for the two highest initial masses.

Of much greater importance, however, is the effect of main-sequence mass loss on the post-main-sequence phases of evolution. Here three competing factors related to mass loss vie with each other, two of which can be recognized from Figure 2 and the third from Figure 1. The first factor is simply the increase of the mass fraction of the helium core, which tends to keep the stellar radius small. The second

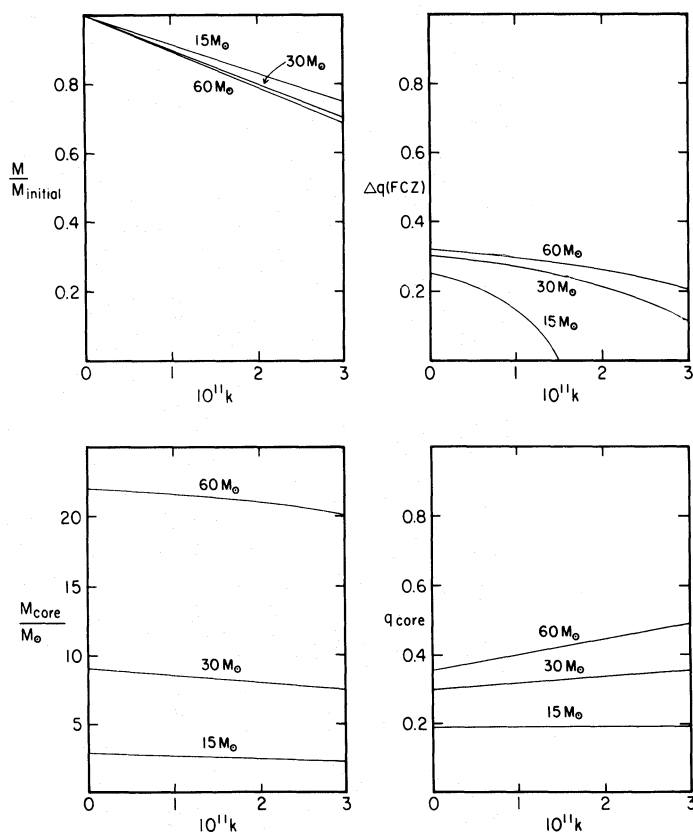


FIG. 2.—Zonal boundaries in stellar models based on the Schwarzschild criterion for convection as a function of the mass-loss parameter k . The stage of evolution shown is that where the FCZ attains its greatest size. Notation is as follows: M , mass of the star; M_{core} , mass of the helium core; q_{core} , mass fraction of the star occupied by the helium core; $\Delta q(\text{FCZ})$, mass fraction of the star occupied by the FCZ.

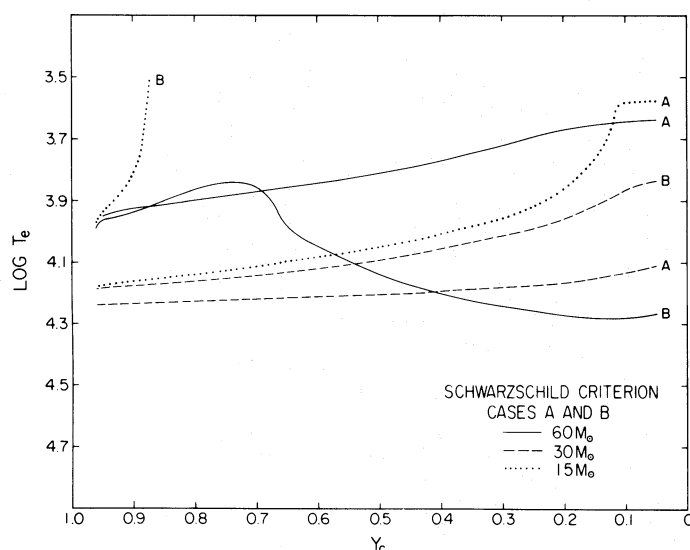


FIG. 3.—Effective temperature versus central helium abundance (during the phase of core helium burning) for stellar models based on the Schwarzschild criterion for convection, according to cases A and B.

factor is the decrease of the mass fraction of the FCZ, which allows the stellar radius to expand. And the third factor is the increase of the luminosity of the star with respect to a star of the same mass that has reached the same central temperature without any mass loss; the higher luminosity in the envelope of the star that has lost mass tends to increase this star's radius. These three factors then determine where the star will settle down in the H-R diagram to burn core helium.

When $k = 1 \times 10^{-11}$, it turns out that only stars that are initially heavier than $\sim 45 M_{\odot}$ will settle down as blue supergiants. Less massive stars become red supergiants. But when $k = 3 \times 10^{-11}$, even the most

massive stars are found to expand immediately to the dimensions of red supergiants.

Once helium depletion begins, mass loss can continue to be important. This fact is illustrated by Figures 1 and 3 for stars that burn core helium as blue supergiants. In § IV, the case of stars burning core helium as red supergiants will be discussed. Since the transitional stages between the end of core hydrogen burning and the onset of the slow stages of core helium burning are always very fast, little mass is lost during this brief interlude.

In addition to the shifted location of the star on the H-R diagram, another clue to a large amount of mass loss is enhanced helium at the stellar surface. None

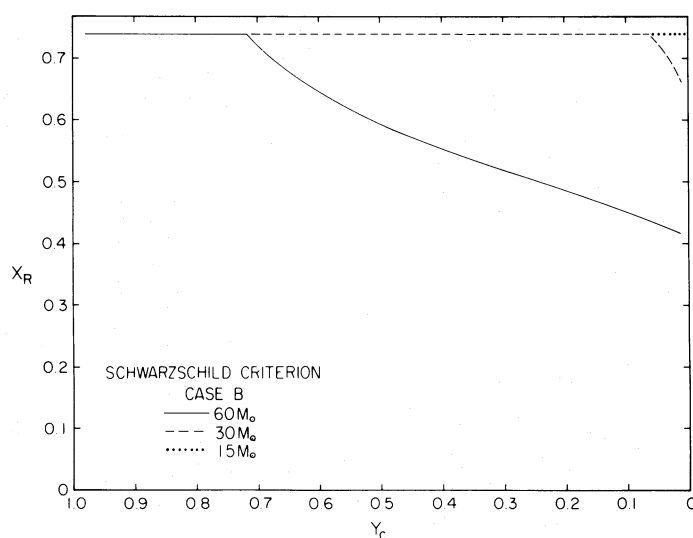


FIG. 4.—Surface hydrogen abundance versus central helium abundance (during the phase of core helium burning) for stellar models based on the Schwarzschild criterion for convection, according to case B.

of our models for $k = 1 \times 10^{-11}$ suffers enough mass loss to exhibit helium richness before attaining the dimensions of a yellow supergiant. Even then, the effect is not large (see Fig. 4). But for $k = 3 \times 10^{-11}$, helium enrichment appears shortly before the end of core hydrogen burning if the star's initial mass exceeds $\sim 50 M_{\odot}$. The reason why only the more massive models show helium enrichment is that their core boundaries lie initially closer (in mass fraction) to the stellar surface; it is not true that the more massive models lose, in any significant way, proportionately more mass than do the models of lower mass (see Fig. 2).

c) Cases C and D

These cases will be discussed in § IVc, d.

IV. EVOLUTION BASED ON THE LEDOUX CRITERION FOR CONVECTION

a) Case A

By adopting the Ledoux criterion for convection, evolution at constant mass was treated by us previously for stars of 15 and $30 M_{\odot}$ (Stothers and Chin 1975). Here, however, we have generated a new set of sequences, including one for $60 M_{\odot}$, because our present choice of the convective mixing length is different from before and has an important effect on the stellar models.

Briefly, the main-sequence phase of evolution is much the same as in the case where the Schwarzschild criterion for convection was adopted, except that the Ledoux criterion leads to a smaller semiconvective zone and one that is usually detached from the convective core. The FCZ that develops after the

stage of central hydrogen exhaustion occurs, in the present models, only for stars more massive than $\sim 35 M_{\odot}$; therefore our only sequence for which core helium burning is initiated in the blue-supergiant configuration is the sequence for $60 M_{\odot}$ (see Fig. 5).

As a surprising result of our assumption that the convective mixing length is proportional to the pressure scale height rather than to the density scale height, our new evolutionary sequences for 15 and $30 M_{\odot}$ do not leave the region of red supergiants at any time during the phase of core helium burning. The only obvious structural differences between the new models and the old ones are related to a large density inversion that appears in the convective envelopes of the new models. To make this inversion smaller, we have rerun the track for $30 M_{\odot}$ with larger values of α_p . This causes the march of density with respect to pressure in the envelope to become more nearly adiabatic and the surface temperature to become hotter, so that the zone where the density inversion appears lies closer to the unimportant surface layers. For a value of $\alpha_p = 10$ the star succeeds in leaving the region of red supergiants. As in the older published sequence, the transition from red to blue occurs when helium in the core is about half depleted. The main difference between the two sequences lies in the average effective temperature of the red supergiants, viz., $\log T_e \approx 3.8$ in the present sequence and $\log T_e \approx 3.6$ in the older sequence.

Nevertheless, the absence of the blue loop for $\alpha_p = 1$ should not be considered definitive. Blue loops are known in general to be very sensitive to minor changes in many of the stellar input parameters (Stothers and Chin 1973a). Thus, Ziolkowski (1972) obtained a blue loop for $\alpha_p = 1$ with other input parameters approximating ours.

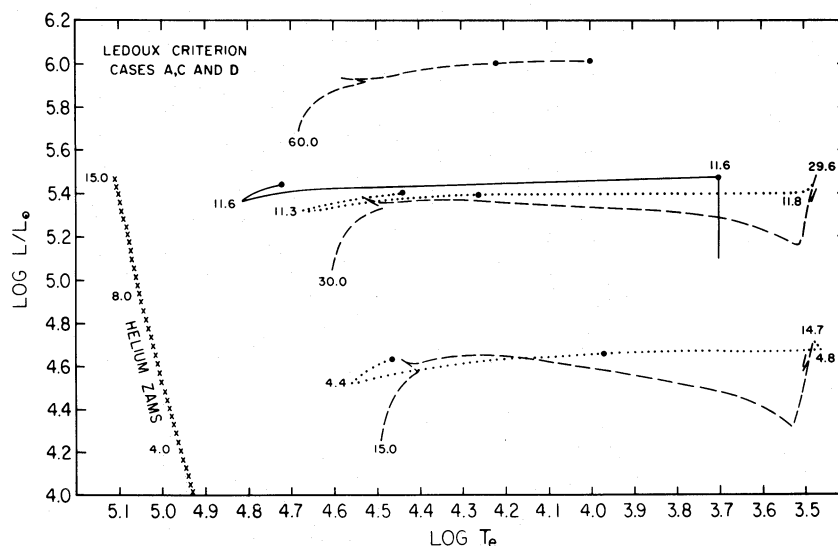


FIG. 5.—H-R diagram showing the evolutionary tracks for stellar models based on the Ledoux criterion for convection, according to case A (dashed lines), case C (dotted lines), and case D (solid line). Heavy dots mark the beginning and end of the slow blue stages of core helium burning. All the tracks terminate shortly before central helium exhaustion. The helium main sequence is shown for reference. Masses are indicated in solar units.

b) Case B

The main-sequence phase of evolution is practically identical to that derived by using the Schwarzschild criterion (§ IIIb), except that, now, the hydrogen-burning lifetime is not significantly affected by the suppression of semiconvection. Subsequent evolutionary developments carry the star directly into the red-supergiant configuration, unless mass loss has been very slight and the initial mass exceeds $\sim 45 M_{\odot}$. For comparison, when the Schwarzschild criterion is adopted, only a substantial amount of mass loss or a large initial metals abundance (Stothers and Chin 1976) will directly produce a red supergiant.

c) Case C

The results obtained for this case, although they are based explicitly on the Ledoux criterion for convection and on the assumption of no mass loss on the main sequence, can be conveniently taken as representing the results for all other cases (regardless of the criterion for convection or the amount of main-sequence mass loss) in which the star attains a red-supergiant configuration during the early stages of core helium burning. The reason for this identity of results is that, as a red supergiant, the star will very quickly lose all but a few percent of its hydrogen envelope if $k \geq 1 \times 10^{-11}$.

A loss of only 10% of the star's mass is enough to suppress any normal blue loop that might otherwise tend to develop (Lauterborn, Refsdal, and Weigert 1971; Lauterborn and Siquig 1974; Siquig and Sonneborn 1976; Sreenivasan and Wilson 1978; Stothers and Chin 1978). Continuing mass loss will bring the percentage to about 65%, whereupon the star becomes so nearly homogeneous in composition that it suddenly shifts to the blue side of the H-R

diagram. The precise time (or central helium abundance) at which it does so depends of course on the prior rate of mass loss. For $k = 1 \times 10^{-11}$, the shift occurs when $Y_c \approx 0.8$ (Fig. 6). No further mass loss takes place thereafter.

The bluest models during the remainder of core helium burning possess the following characteristics, for respective initial masses of 15 and $30 M_{\odot}$: total mass, 4.4 and $11.3 M_{\odot}$; mass of the residual hydrogen envelope, 10% and 8% of the total mass; surface hydrogen abundance X_R , 0.71 and 0.59; and central helium abundance Y_c , 0.20 and 0.45. These objects lie very close to the normal ZAMS line in the H-R diagram (Fig. 5) but are differentiated from the normal stars by their exceptionally low masses.

If the extent of mass loss from the red supergiants is less than 10%, a normal blue loop (rather than one induced by removal of the hydrogen envelope) may develop. However, the bluest point along this loop is always redder than in the case just discussed, and becomes still redder as the amount of mass loss increases (Lauterborn, Refsdal, and Weigert 1971; Sreenivasan and Wilson 1978; Stothers and Chin 1978). Therefore, no confusion with main-sequence stars should arise in this situation.

d) Case D

The physical cause underlying case D is an atmospheric density inversion which occurs in yellow supergiants more massive than $\sim 20 M_{\odot}$ and which may induce very rapid mass loss (Peterson 1971; Bisnovatyi-Kogan and Nadezhin 1972; Schmid-Burgk and Scholz 1975). Our sequence for $60 M_{\odot}$ remains too blue to encounter this instability, but not our sequence for $30 M_{\odot}$, which suffers a sudden drop in mass to

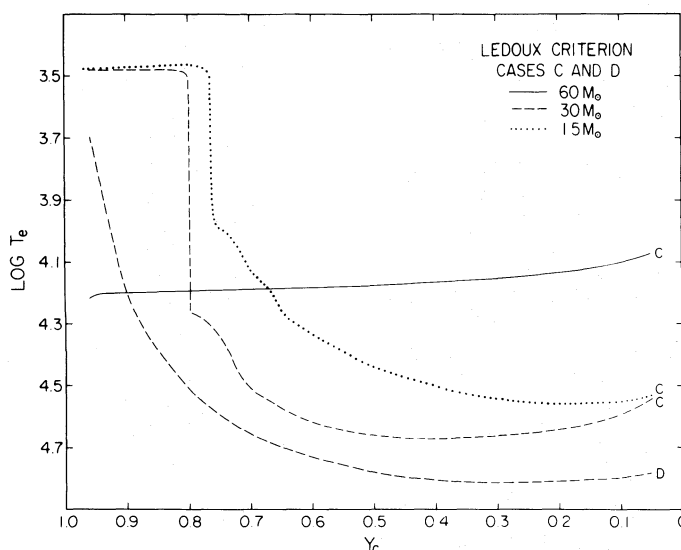


FIG. 6.—Effective temperature versus central helium abundance (during the phase of core helium burning) for stellar models based on the Ledoux criterion for convection, according to cases C and D.

11.6 M_{\odot} , when, following our prescription, $\log T_e = 3.7$. The star at this time is in a transitional stage preceding steady core helium burning, and fluctuates noticeably in luminosity before equilibrium is achieved (see Fig. 5).

The onset of the slow stages of core helium burning marks the point in Figure 5 where the star breaks away from the effective-temperature barrier and makes a long passage to $\log T_e \approx 4.8$. No further mass loss is assumed. At its bluest stage, the star has only 7% of its mass in its residual hydrogen envelope and composition values of $X_R = 0.39$ at the surface and $Y_c = 0.29$ at the center. Thus it resembles the remnant derived for case C.

The evolutionary sequence for 30 M_{\odot} computed by Bisnovaty-Kogan and Nadezhin (1972) is very similar to our sequence as far as their calculations went (only to a mass of 22.7 M_{\odot}). Sequences corresponding to a case somewhere between our cases D and C were computed for 20 and 40 M_{\odot} by Chiosi, Nasi, and Sreenivasan (1978). In their work, the remnant of the more massive star preserves a thick hydrogen envelope, containing 22% of the final mass, and so is understandably much cooler at the surface (by $\Delta \log T_e \approx 0.5$) than is our model. The remnant of their less massive star, although it has a hydrogen envelope of only 5% of the final mass, is as cool as their other remnant. The reason for such a low effective temperature is unclear; however, Chiosi, Nasi, and Sreenivasan mention some difficulty in computing the final models along their evolutionary tracks.

V. VARIOUS EVOLUTIONARY HYPOTHESES CONCERNING THE MOST MASSIVE STARS

As noted in § I, there exists observational evidence that the stars of greatest mass never achieve very large radii. At least five ways can be suggested by which a star may limit its radius.

One way is by putting the star in a close binary system, so that the Roche lobe of the star sets an upper limit to its possible size. However, not all O stars appear to be in binary systems (Conti, Leep, and Lorre 1977; Bolton and Rogers 1978).

A second way of limiting the radius is by suppressing convection in the core, say, by an intense magnetic field (Stothers and Chin 1973*b*), or by restricting the convective core to a very small region near the center, say, by means of a semiconvective composition gradient (scheme S1 in Stothers 1970). However, a variety of observational and theoretical evidence strongly opposes the idea of a negligible amount of convection in the cores of upper-main-sequence stars (see the papers just cited).

A third way of maintaining a small radius is by mixing core material well beyond the formal convective core boundary (schemes C1 and H in Stothers 1970). This situation may come about by convective overshooting from the core or by rotationally induced currents. If the whole star is kept in a chemically homogeneous state, the radius will actually shrink as time goes on.

This limiting case will be calculated here in an entirely general way. We begin by computing a series of ZAMS models with differing values of the hydrogen abundance. These models are plotted in Figure 7. Lines connecting points for the same stellar mass represent the conservative mode of evolution, while evolution with loss of mass, starting from 120 M_{\odot} , can take the star to any point in the manifold of lower (M, X) values, depending on the rate of mass loss. Observable stars, in this picture, ought always to lie to the left of the ZAMS, as it is normally defined.

The fourth way of avoiding a large radius is by assuming such a high rate of mass loss that chemical inhomogeneities, when they develop, remain always small. Results based on this assumption are shown in Figures 8 and 9 for a star of initially 120 M_{\odot} . The qualitative resemblance of these results to earlier results determined, more crudely, by Simon and Stothers (1970) is surprisingly good.

If a value of $k = 10 \times 10^{-11}$ is adopted, the rate of mass loss reaches $3 \times 10^{-5} M_{\odot} \text{ yr}^{-1}$, and the star evolves eventually to the left of the ZAMS when $X_c = 0.25$. With a rate 3 times higher, the star is forced to evolve down the ZAMS. In this case, the star's average hydrogen abundance X is 0.59, 0.46, 0.39, and 0.36 when its mass is respectively 60, 30, 15, and 7 M_{\odot} . This scheme of evolution is reminiscent of the scheme proposed by Russian astronomers two decades ago (e.g., Fesenkov and Idlis 1959; Masevich 1959). The suggestion that O3 stars lie to the left of the ZAMS (Conti and Burnichon 1975) must remain untested until the effective-temperature scale for O stars is settled (see Stothers 1976).

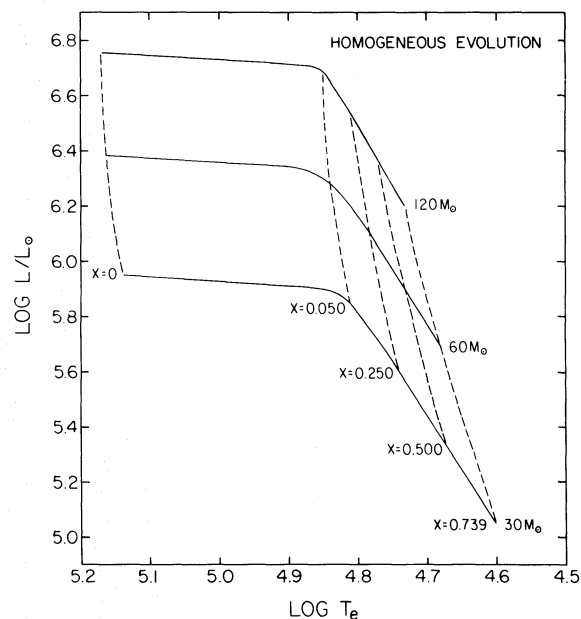


FIG. 7.—H-R diagram showing the evolutionary tracks for homogeneous (completely mixed) stellar models of 30, 60, and 120 M_{\odot} , from the normal ZAMS to the helium main sequence. Mass loss has been neglected. At several stages, the hydrogen abundance X is plotted.

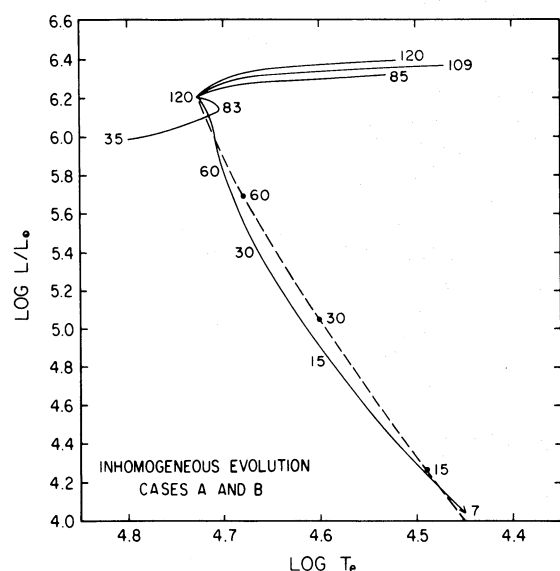


FIG. 8.—H-R diagram showing the evolutionary tracks for inhomogeneous stellar models of initially $120 M_{\odot}$, based on the Schwarzschild criterion for convection. The tracks cover the main phase of core hydrogen burning for cases A and B. In descending order of luminosity, the tracks represent $k = 0$, 1×10^{-11} , 3×10^{-11} , 10×10^{-11} , and 30×10^{-11} . The ZAMS line (dashed) can be regarded as the track for $k = \infty$. Masses are indicated in solar units.

A fifth possible way of keeping the star's radius small requires a very high mass and essentially no mass loss, so that the FCZ that develops after central hydrogen exhaustion becomes as large as possible. In a previous paper (Stothers and Chin 1976) we showed

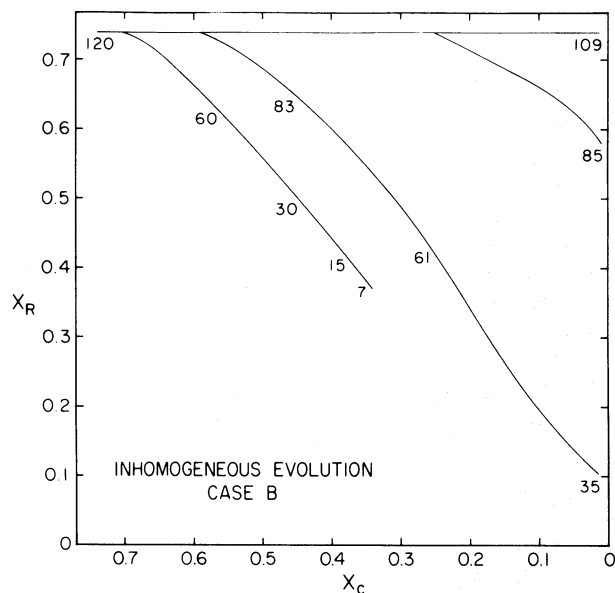


FIG. 9.—Surface hydrogen abundance versus central hydrogen abundance for inhomogeneous stellar models of initially $120 M_{\odot}$, based on the Schwarzschild criterion for convection. The curves refer to case B and can be identified from Fig. 8. Masses are indicated in solar units.

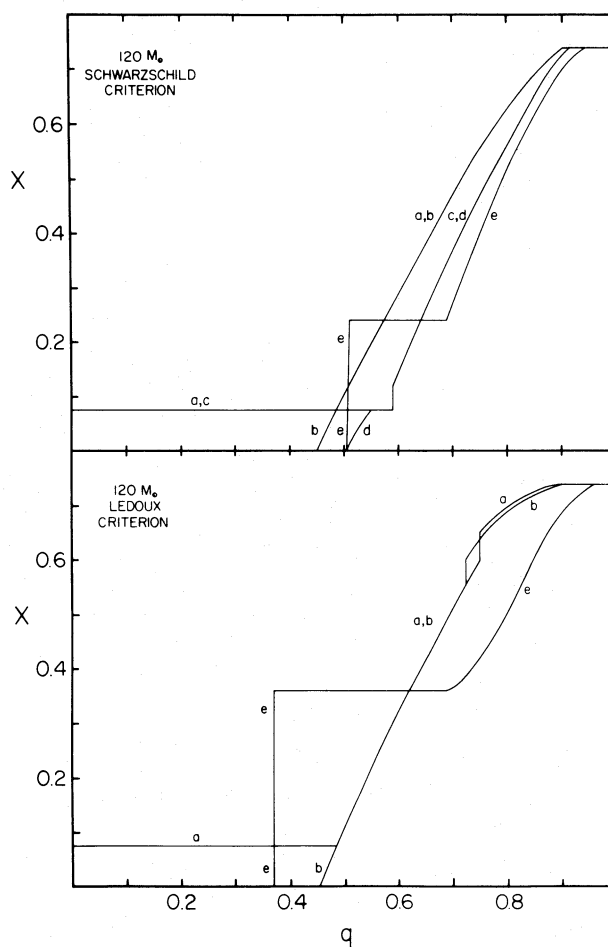


FIG. 10.—Hydrogen profiles in inhomogeneous stellar models of $120 M_{\odot}$, based on the Schwarzschild and Ledoux criteria for convection. Mass loss has been neglected. Lettering index: *a*, stage when $X_c = 0.08$ for the first time; *b*, end of the original phase of core hydrogen burning; *c*, merger of the FCZ and the convective core; *d*, end of the second phase of core hydrogen burning; and *e*, maximum extent of the FCZ during core helium burning.

that the FCZ in a sequence for $60 M_{\odot}$ approaches to within 1 density scale height of the convective core. At this mass, the degree of convective overshooting between the two unstable zones is probably small. But at $120 M_{\odot}$ we find that the two zones actually merge, provided that the Schwarzschild criterion for convection is adopted. Thus, in a matter of only weeks, the center of the star is transformed from a completely dehydrogenized state back into a state with $X_c = 0.08$ (see Fig. 10).

Although it would seem, from the foregoing considerations, that a star of initially $120 M_{\odot}$ could hardly avoid keeping a small size, this conclusion is actually unwarranted. Further evolution of the inhomogeneous models without mass loss is found to take them (after the final exhaustion of central hydrogen) into the region of red supergiants when the Schwarzschild criterion for convection is adopted.

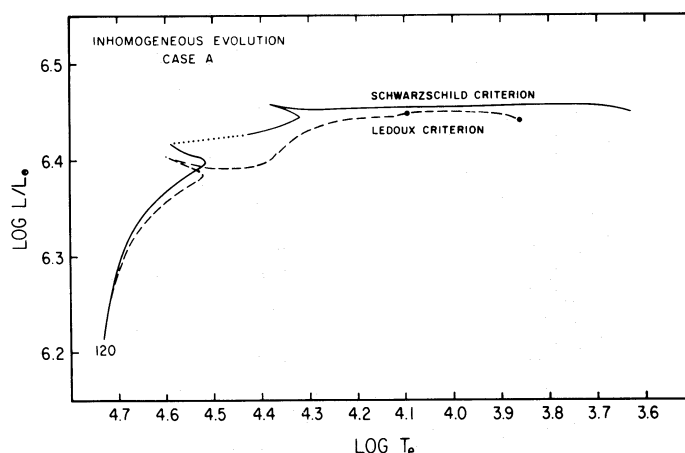


FIG. 11.—H-R diagram showing the evolutionary tracks for inhomogeneous stellar models of $120 M_{\odot}$, based on the Schwarzschild and Ledoux criteria for convection. Mass loss has been neglected. The dotted portion of the track for the Schwarzschild criterion represents the phase of evolution when mixing occurs between the FCZ and the convective core. Heavy dots on the track for the Ledoux criterion mark the beginning and end of the slow stages of core helium burning.

When the Ledoux criterion is adopted, the models settle down in the region of blue and yellow supergiants (Fig. 11).

VI. COMPARISON WITH OBSERVATIONS

a) Mass-Loss Rates

How realistic are our assumed rates of mass loss? In Figure 12 the rates used in the present evolutionary sequences for case B with $k = 1 \times 10^{-11}$ are plotted as individual contour lines (units of $10^{-7} M_{\odot} \text{ yr}^{-1}$)

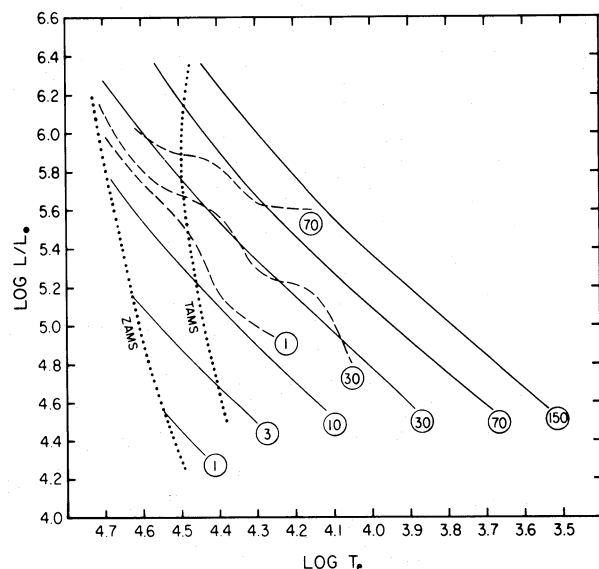


FIG. 12.—Rates of mass loss (units of $10^{-7} M_{\odot} \text{ yr}^{-1}$) are plotted on the H-R diagram for stellar models based on the Schwarzschild criterion for convection that are losing mass according to case B with $k = 1 \times 10^{-11}$. The contour lines (solid) connect points for the same rate of mass loss as taken from the evolutionary tracks in Fig. 1. Hutchings's observed rates for early-type supergiants are also shown (dashed lines).

across the H-R diagram. Also indicated, in the same manner, are Hutchings's (1976) average observed mass-loss rates for O and B supergiants; these average rates fall within half an order of magnitude of the rates determined independently by Barlow and Cohen (1977). Hutchings's rates seem to follow fairly well the McCrea (1962) law, $-dM/dt \propto LR/M$, although Barlow and Cohen's rates more closely follow the Fesenkov (1949) law, $-dM/dt \propto L$.

Despite the considerable amount of observational uncertainty, it seems most likely that our assumed rates of mass loss (with $k \geq 1 \times 10^{-11}$) are unrealistically large for the main-sequence phase of evolution, yet are consistent with the available observational data for at least the early post-main-sequence stages. Therefore, the relevant evolutionary tracks to adopt would be those for case A (unless the mass significantly exceeds $\sim 30 M_{\odot}$), up to the beginning of the slow stages of core helium burning.

Since core helium burning may occur over a wide range of effective temperatures, mass loss at that time could be highly variable. Our case B probably exaggerates the rate of mass loss for blue supergiants with $\log T_e < 4.1$, since the rates observed for these effective temperatures are either constant (Barlow and Cohen 1977) or relatively small (Lamers, Stalio, and Kondo 1978). On the other hand, at very low effective temperatures, the observed rates rise again. Within an order of magnitude, they range from 10^{-7} to $10^{-5} M_{\odot} \text{ yr}^{-1}$, the higher rates characterizing brighter stars (e.g., Kudritzki and Reimers 1978; Hagen 1978). But rates of at least 5×10^{-5} to $3 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$ would actually be needed if red supergiants of respectively 15 to $30 M_{\odot}$ are to lose a significant amount of mass during their helium-burning lifetimes.

b) Blue Supergiants

Theory predicts the existence of a zone in the H-R diagram where stars are expected to evolve rapidly

between the last stages of core hydrogen burning and the beginning of the slow stages of core helium burning. Observationally, this zone should appear as a poorly populated area between the main-sequence band and the region occupied by most blue supergiants. However, no gap of this type is actually observed among stars more massive than $\sim 15 M_{\odot}$.

One suggestion for filling this gap is to invoke mass loss. Since a *moderate* amount of mass loss makes matters worse by widening the gap (Fig. 1), heavy mass loss in some post-main-sequence phase of evolution seems to be necessary (Fig. 5). Yet the amount of mass loss that is required seems unrealistic from at least three points of view. First, the directly observed rates of mass loss from blue and red supergiants (at least for initial masses up to $\sim 30 M_{\odot}$) appear to be much too small. Second, the indirectly observed masses of blue supergiants, however flimsily determined (Stothers 1972; de Loore, De Grève, and Lamers 1977), do not seem to be lower by the necessary factor of 3 than the masses expected for the case of no mass loss. And third, blue supergiants are not observed to be particularly helium-rich.

As an alternative explanation, a certain degree of cosmic scatter in the initial chemical composition could be invoked. A spread in X_e or Z_e would widen both the main-sequence band of stars burning core hydrogen and the supergiant band of stars burning core helium, so that the gap between the two could, in principle, be bridged. However, a very unusual hydrogen or metals abundance would have to exist in some stars, because the width of the gap can be reduced to only $\delta \log T_e \approx 0.10$ if one assumes a reasonable range of initial chemical compositions, $X_e = 0.60$ – 0.74 and $Z_e = 0.02$ – 0.04 .

c) Red Supergiants

No red supergiants are observed to be significantly brighter than $\log(L/L_{\odot}) \approx 5.3$. Therefore, direct information on their rates of mass loss at higher luminosities is lacking. However, at lower luminosities, empirically determined masses of red supergiants seem to be entirely normal (Stothers and Leung 1971; Cowley, Hutchings, and Popper 1977), in conformity with the moderate rates of mass loss that are observed. If we accept these observed rates, fit them to equation (1) (following Kudritzki and Reimers 1978), and predict the rates for red supergiants that are much more luminous, the amount of mass loss is still found to be unimportant from an evolutionary standpoint.

Therefore, the absence of very luminous red supergiants must be explained in one of the following three ways. First, heavy mass loss may occur at some earlier stage of evolution, and prevent a star from ever attaining the dimensions of a red supergiant. Second, the development of a large FCZ in the envelope of a massive star just after the stage of central hydrogen exhaustion will keep the star blue during the subsequent phase of core helium burning; however, some mechanism has to be invoked to halt the envelope expansion after this phase. Mass loss, occurring at a

rate far greater than that expected from a simple extrapolation of the observed rates, is one possibility. An alternative possibility is copious neutrino emission, which so shortens the remaining stellar lifetime that very few supergiants would ever be seen in such an evolved state (e.g., Stothers and Chin 1970).

If mass loss is the correct explanation, then case C (or B) is much more likely than case D. In case D, no red supergiants are expected to be found more massive than $\sim 20 M_{\odot}$, whereas observed red supergiants range up to $\sim 30 M_{\odot}$.

d) OBN and WN Stars

Some O and B stars, as well as some Wolf-Rayet stars, show a strong enhancement of their nitrogen lines. This is commonly interpreted as an abundance effect. To produce such an anomaly by mass loss, the hydrogen-processed interior of the star must be exposed through removal of about 40% of the initial mass (see especially Dearborn and Eggleton 1977). However, we have argued above that such a large amount of mass loss is unlikely to have taken place from ordinary O and B stars.

Perhaps a more realistic expectation is that only the most massive O stars of all are reduced to such a nitrogen-rich state by mass loss. If so, the WN stars with their large nitrogen and helium overabundances and their bright luminosities are the most suitable candidates (Rublev 1965; Tanaka 1966*b*; Simon and Stothers 1970; Bisnovatyi-Kogan and Nadezhin 1972; Conti 1976; Dearborn *et al.* 1978; Chiosi, Nasi, and Sreenivasan 1978; de Loore, De Grève, and Vanbeveren 1978; Stothers and Chin 1978). These stars are placed on the H-R diagram in Figure 13, where the observational data are taken from Smith (1973) for all but the WN7 stars, whose effective temperatures are given by Conti (1976). Conti assumed a mean effective temperature of $\sim 43,000$ K, but Seggewiss and Moffat (1979) have given $> 33,000$ K, while Morton (1970) estimated $\sim 30,000$ K and Schild (1968) a value close to that of the B0 supergiant ϵ Ori, i.e., $\sim 25,000$ K. If Schild's suggestion is accepted, the bolometric corrections of the WN7 stars become much smaller in absolute value, and therefore the assigned luminosities become fainter; consequently, the stars end up in the H-R diagram as merely a cool extension of the sequence of other WN stars.

Although, for this reason, the WN7 stars cannot be regarded as necessarily having arisen from the initially most massive stars, nevertheless all the WN stars could have evolved into their present state through the operation of a stellar wind according to any of our cases B, C, or D. Those WN stars that are members of close binary systems could also have lost their hydrogen envelopes through the process of mass transfer to their companions (Paczynski 1967). Of these possibilities, case B runs into the problem that the observed rates of mass loss from Of stars (presumably the precursors of WN stars) average only 10^{-6} to $10^{-5} M_{\odot} \text{ yr}^{-1}$ (Lamers and Morton 1976; Hutchings 1976; Barlow and Cohen 1977; Conti and Frost 1977),

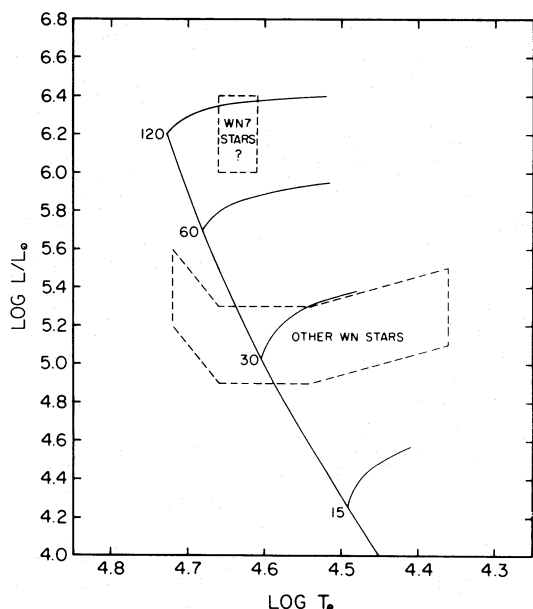


FIG. 13.—H-R diagram showing the zones where observed WN stars are located. Theoretical evolutionary tracks for case A are added for reference, with the input masses indicated in solar units.

corresponding to our case B with $k \approx 3 \times 10^{-11}$. A glance at Figures 1 and 8 shows that rates this low will not immediately produce from main-sequence stars hot remnants like WN stars. Cases C and D also suffer from serious defects which have already been discussed in the preceding subsection. One main difference in the status of the remnants expected for the different cases is that, in case B, a WN star could still be burning core hydrogen, while, in cases C and D, it would have to be burning core helium or heavier elements.

A more general problem with these theories is the low observed effective temperatures of WN5 stars, which show no hydrogen at their surfaces (Smith 1973) but are apparently much cooler than the theoretical helium main sequence (compare Fig 5; also Stothers 1976). One way to resolve this problem is to argue that hydrogen is in fact present but difficult to observe (Underhill 1973). Another way is to revise upward the observed effective temperatures of these stars. An interesting consequence of applying the latter remedy to other WN stars is that the true range of their luminosities would be greater than indicated in Figure 13, so that their initial masses could actually be 20–120 M_{\odot} , instead of only values concentrated in the suspiciously narrow range 20–40 M_{\odot} , as implied by the figure. Clearly, more observations are needed.

VII. CONCLUSIONS

The effect of a stellar wind on the evolution of stars in the mass range 15–120 M_{\odot} has been investigated in the present paper. Four possible cases of mass loss have been distinguished: A, no mass loss at all; B, substantial mass loss from stars in all stages of

evolution; C, heavy mass loss from red supergiants only; and D, sudden and very heavy mass loss from luminous yellow supergiants. All the stellar models in the present investigation have been constructed with the use of Cox-Stewart opacities.

The assumption of mass loss during the main-sequence phase of evolution is found to lead to a lowering of the luminosity and, unless the mass loss is extremely heavy, of the effective temperature as well. The drop in luminosity and the associated damping of semiconvection have significant, though opposing, effects on the hydrogen-burning lifetime. When hydrogen is exhausted at the center of the star, the growth of a fully convective zone (FCZ) immediately above the hydrogen-burning shell is found to be inhibited by the loss of mass, so that the stellar envelope may be able to expand to a large radius, regardless of the adopted criterion for convection. However, if mass loss is slight, the size of the FCZ depends strongly on which criterion for convection is adopted. In this case, the star settles down to burn core helium as either a blue or a red supergiant, depending on whether the Schwarzschild or Ledoux criterion is adopted as well as on what initial metals abundance and convective mixing length are assumed. But a higher rate of main-sequence mass loss invariably produces only a red supergiant during core helium burning (as a result of the suppression of the FCZ), while a very high rate produces only a very blue helium star (as a result of the removal of most of the hydrogen envelope).

A comparison of the adopted rates of mass loss with the observed rates suggests that stellar winds are probably not an important factor in the evolution of main-sequence stars and supergiants unless the initial masses are greater (perhaps significantly greater) than $\sim 30 M_{\odot}$. On the other hand, in order to account by mass loss for the apparent nitrogen richness of the OBN and WN stars and for the continuous population of bright supergiants between spectral types O and A, a high rate of mass loss is required for initial stellar masses at least down to $\sim 15 M_{\odot}$. The most likely site for mass loss at these modest stellar masses is the red-supergiant region (case C of mass loss). With a high assumed rate of mass loss, the remnants will eventually appear as very blue stars, resembling supergiants of normal luminosity on the H-R diagram but having abnormally low masses and high surface helium and nitrogen abundances. Together with the normal blue supergiants (the immediate progenitors of the red supergiants), these remnants will constitute a blue group whose total population will obviously depend on the particular times at which the stars enter and leave the red-supergiant configuration. In this connection, the likelihood of catastrophic mass loss from luminous yellow supergiants (case D) would seem to be small, because some red supergiants are observed to have masses significantly higher than the predicted cutoff mass of $\sim 20 M_{\odot}$.

Among the most massive stars of all, a loss of two-thirds of the initial mass on the main sequence could account for the WN7 stars and for the lack of

very luminous supergiants later than B3. But an alternative interpretation is that this amount of mass loss occurs when the star is (very temporarily) a yellow or red supergiant. The absence of very luminous red supergiants could also be due to the very short lifetime of the star under the influence of neutrino emission, even if the star remained red after core helium burning. Certainly the presupernova state of the star will depend critically on the amount of mass lost.

It is clear that the assumption of mass loss leads to certain predictions that are observationally favored, but to others that are contradictory or, at best, ambiguous. Chiosi, Nasi, and Sreenivasan (1978) have

also emphasized these uncertainties. There seem to be too many free parameters in the stellar models at the present time. In addition to the mass-loss rate, the stellar opacity is a still uncertain quantity. The merits of the Cox-Stewart and Carson opacities in accounting for the observations of massive stars have already been weighed elsewhere (Stothers 1976; Stothers and Chin 1978). Our present results provide a more solid base for comparison, but they do not alter the essential conclusion arrived at earlier, namely, that neither set of opacities can account adequately for all the available observations. However, the fault could still lie in some other form of incompleteness of the stellar models.

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